MATH 223.3 2006-2007 (01) (03) (05)

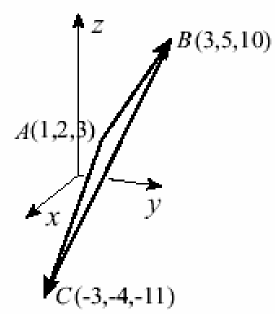
QUIZ #3 - Solutions
Each question is worth 5 points - total = 25.
Integer scores *only*.

#1

The area of the triangle is

$$\frac{1}{2}|\mathbf{AB} \times \mathbf{AC}| = \frac{1}{2}\left\|\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 3 & 7 \\ -4 & -6 & -14 \end{vmatrix}\right\|$$
$$= \frac{1}{2}|(0,0,0)| = 0.$$

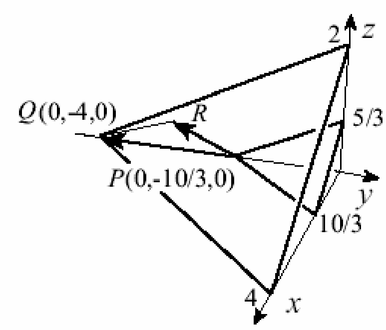
The points must be collinear.



#2

The required distance d is the component of $\mathbf{PQ}$ along $\mathbf{PR}$. Since a vector in the same direction as $\mathbf{PR}$ is $(1,-1,2)$,

$$d = \mathbf{PQ} \cdot \widehat{\mathbf{PR}}$$
$$= (0,-2/3,0) \cdot \frac{(1,-1,2)}{\sqrt{1+1+4}} = \frac{2}{3\sqrt{6}}.$$



#3

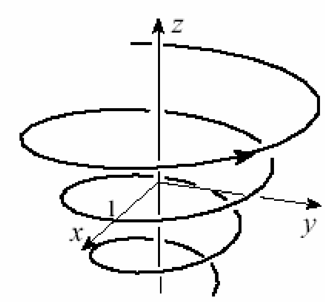
With $\dot{\mathbf{r}} = (e^t\cos t - e^t \sin t, e^t \sin t + e^t\cos t, 1)$ and $\ddot{\mathbf{r}} = (-2e^t \sin t, 2e^t\cos t, 0)$, we obtain

$$\dot{\mathbf{r}} \times \ddot{\mathbf{r}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ e^t(\cos t - \sin t) & e^t(\sin t + \cos t) & 1 \\ -2e^t \sin t & 2e^t \cos t & 0 \end{vmatrix} = (-2e^t\cos t, -2e^t \sin t, 2e^{2t}).$$

Consequently,

$$\kappa(t) = \frac{|\dot{\mathbf{r}} \times \ddot{\mathbf{r}}|}{|\dot{\mathbf{r}}|^3} = \frac{\sqrt{4e^{2t}\cos^2 t + 4e^{2t}\sin^2 t + 4e^{4t}}}{[(e^t\cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2 + 1]^{3/2}}$$
$$= \frac{2e^t\sqrt{1+e^{2t}}}{(1+2e^{2t})^{3/2}},$$

and $\rho(t) = \dfrac{1}{\kappa} = \dfrac{(1+2e^{2t})^{3/2}}{2e^t\sqrt{1+e^{2t}}}$.



#4

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2t\hat{\mathbf{i}} + 2e^t(t+1)\hat{\mathbf{j}} - \frac{2}{t^3}\hat{\mathbf{k}}; \quad |\mathbf{v}| = \sqrt{4t^2 + 4e^{2t}(t+1)^2 + 4/t^6} = 2\sqrt{t^2 + e^{2t}(t+1)^2 + 1/t^6};$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 2\hat{\mathbf{i}} + 2e^t(t+2)\hat{\mathbf{j}} + \frac{6}{t^4}\hat{\mathbf{k}}$$

#5

If (x,y) is made to approach $(1,1)$ along the straight lines $y-1 = m(x-1)$,

$$\lim_{(x,y)\to(1,1)} \frac{x^2-2x-y^2+2y}{x^2-2x+y^2-2y+2} = \lim_{x\to 1}\frac{(x-1)^2-(y-1)^2}{(x-1)^2+(y-1)^2} = \lim_{x\to 1}\frac{(x-1)^2 - m^2(x-1)^2}{(x-1)^2+m^2(x-1)^2}$$
$$= \lim_{x\to 1}\frac{1-m^2}{1+m^2} = \frac{1-m^2}{1+m^2}.$$

Since this result depends on m, the original limit does not exist.